Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2009

MARKING SCHEME

APPLIED MATHEMATICS

ORDINARY LEVEL

General Guidelines

1. Penalties of three types are applied to a candidate's work as follows:

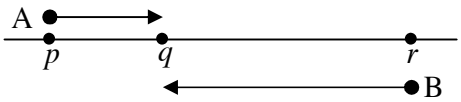
Slips	– numerical slips	S (–1)
Blunders	– mathematical errors	B (–3)
Misreading	– if not serious	M (–1)

2. An 'attempt mark' is awarded as follows: 5 (att 2) 10 (att 3)

   Where a serious blunder or omission or misreading oversimplifies the calculations, then an 'attempt mark' is only awarded.

3. The marking scheme shows one correct solution to each question.
   In many cases there are other equally valid methods.

1. 3 points p, q and r lie on a straight level road.



Two cars, A and B, are moving towards each other on the road.

Car A passes p with speed 3 m/s and uniform acceleration of 2 m/s^2 and at the same instant car B passes r with speed 5 m/s and uniform acceleration of 4 m/s^2.

A and B pass each other at q seven seconds later.

Find **(i)** the speed of car A and the speed of car B at q.

(ii) $|pq|$ and $|rq|$, the distances A and B have moved in these 7 s.

Car A stops accelerating at q and continues on to r at uniform speed.

(iii) Find, correct to one place of decimals, the total time for car A to travel from p to r.

(i)

$$v = u + at$$
$$v_A = 3 + 2(7)$$
$$v_A = 17 \text{ m/s.}$$ 10

$$v = u + at$$
$$v_B = 5 + 4(7)$$
$$v_B = 33 \text{ m/s.}$$ 10

(ii)

$$s = ut + \tfrac{1}{2}at^2$$
$$s_A = 3(7) + \tfrac{1}{2}(2)(49)$$
$$s_A = 70 \text{ m.}$$ 10

$$s = ut + \tfrac{1}{2}at^2$$
$$s_B = 5(7) + \tfrac{1}{2}(4)(49)$$
$$s_B = 133 \text{ m.}$$ 10

(iii)

$$s = ut + \tfrac{1}{2}at^2$$
$$133 = 17(t) + 0$$
$$t = 7.8 \text{ s.}$$

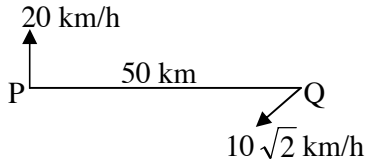
total time = 14.8 s. 10

50

2. A ship P is moving north at a constant speed of 20 km/h.

Another ship Q is moving south-west at a constant speed of $10\sqrt{2}$ km/h.

At a certain instant, P is positioned 50 km due west of Q.



Find **(i)** the velocity of P in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of Q in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of P relative to Q in terms of $\vec{i}$ and $\vec{j}$

(iv) the shortest distance between P and Q in the subsequent motion.

(i) $\vec{V}_P = 0\,\vec{i} + 20\,\vec{j}$ — 10

(ii) $\vec{V}_Q = -10\sqrt{2}\cos 45\,\vec{i} - 10\sqrt{2}\sin 45\,\vec{j}$ — 5

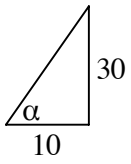
$= -10\,\vec{i} - 10\,\vec{j}$ — 5

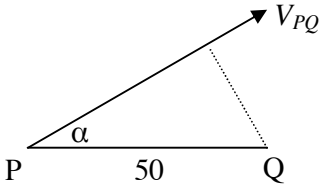
(iii) $\vec{V}_{PQ} = \vec{V}_P - \vec{V}_Q$ — 5

$= \left(0\,\vec{i} + 20\,\vec{j}\right) - \left(-10\,\vec{i} - 10\,\vec{j}\right)$

$= 10\,\vec{i} + 30\,\vec{j}$ — 5

(iv) $\tan\alpha = \frac{30}{10} = 3$ — 5





shortest distance $= 50\sin\alpha$ — 5

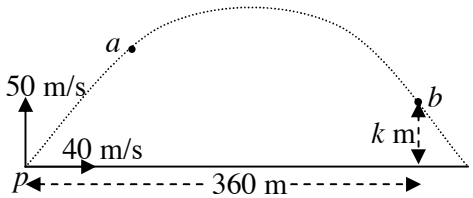
$= 50\sin(71.5651)$ — 5

$= 47.43$ km. — 5

50

3. **(a)** A particle is projected with initial velocity $40\ \vec{i} + 50\ \vec{j}$ m/s from point p on a horizontal plane.

a and b are two points on the trajectory (path) of the particle.



The particle reaches point a after 2 seconds of motion.

The displacement of point b from p is $360\ \vec{i} + k\ \vec{j}$ metres.

Find **(i)** the velocity of the particle at a in terms of $\vec{i}$ and $\vec{j}$

(ii) the speed and direction of the particle at a

(iii) the value of k.

(a) (i)
$$\vec{V} = 40\,\vec{i} + \{50 - 10(2)\}\,\vec{j}$$
$$= 40\,\vec{i} + 30\,\vec{j}$$ — 10

(ii)
$$\text{speed} = \sqrt{(40)^2 + (30)^2}$$
$$= 50 \text{ m/s}$$ — 5

$$\tan\theta = \frac{30}{40}$$
$$\theta = 36.87°$$ — 5

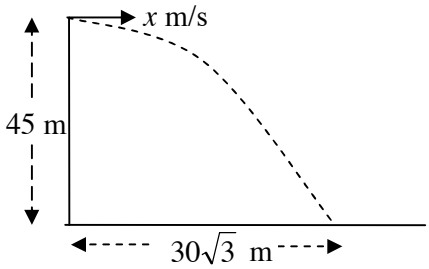
(iii)
$$s_x = ut + \tfrac{1}{2}at^2$$
$$360 = 40(t) + 0$$
$$t = 9 \text{ s}$$ — 5

$$s_y = ut + \tfrac{1}{2}at^2$$
$$k = 50(9) - 5(81)$$
$$k = 45$$ — 5

30

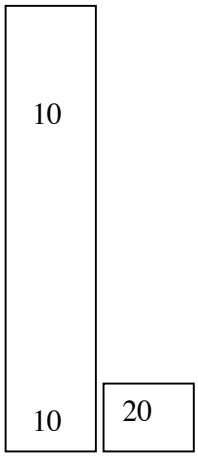
3 **(b)** A straight vertical cliff is 45 m high.
A projectile is fired horizontally with an initial speed of x m/s from the top of the cliff.
It strikes the level ground at a distance of $30\sqrt{3}$ m from the foot of the cliff.

Find the value of x, correct to one decimal place.

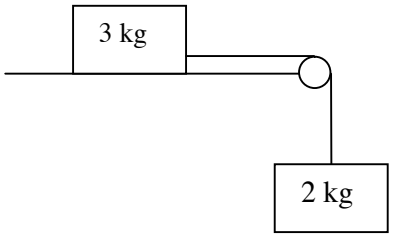


$$s_y = ut + \tfrac{1}{2}at^2$$
$$-45 = 0 - 5t^2$$
$$t = 3$$

$$s_x = ut + \tfrac{1}{2}at^2$$
$$30\sqrt{3} = x(\mathrm{t}) + 0$$
$$30\sqrt{3} = x(3)$$
$$x = 10\sqrt{3}$$
$$= 17.3$$



4. **(a)** Two particles of masses 3 kg and 2 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley at the edge of a smooth horizontal table.
The system is released from rest.

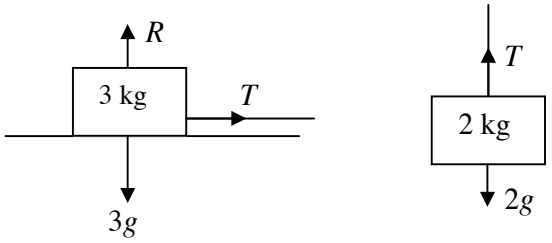


(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the particles.

(iii) Find the tension in the string.

(i)



(ii)

$$T = 3a$$

$$2g - T = 2a$$

$$a = \frac{20}{5} = 4 \text{ m/s}^2$$

(iii)

$$T = 3a$$

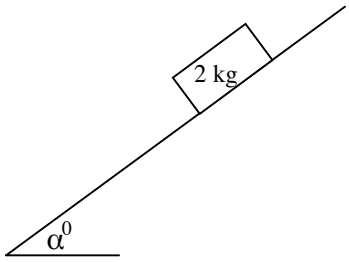
$$T = 12 \text{ N}$$

10	
5	
5	
5	
5	30

4 **(b)** A particle of mass 2 kg is released from rest and slides down a rough plane which is inclined at an angle α^0 to the horizontal,

where $\tan\alpha = \frac{4}{3}$.

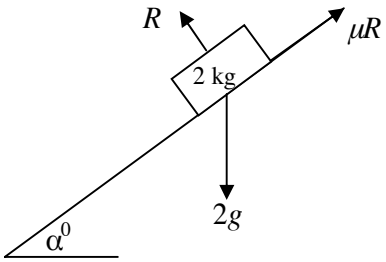
The coefficient of friction between the particle and the plane is $\frac{1}{2}$.



(i) Show on a diagram the forces acting on the particle.

(ii) Find the acceleration of the particle.

(i)



10

(ii)

$$2g\sin\alpha - \mu R = 2a$$

or

$$R = 2g\cos\alpha = 12$$

$$20\left(\frac{4}{5}\right) - \frac{1}{2}(12) = 2a$$

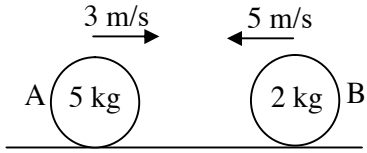
$$a = 5 \text{ m/s}^2$$

5

5

20

5. A smooth sphere A, of mass 5 kg, collides directly with another smooth sphere B, of mass 2 kg, on a smooth horizontal table.



Before impact A and B are moving in opposite directions with speeds 3 m/s and 5 m/s, respectively.

The coefficient of restitution for the collision is $\frac{3}{4}$.

Find **(i)** the speed of A and the speed of B after the collision

(ii) the loss in kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to B due to the collision.

(i) PCM $5(3) + 2(-5) = 5v_1 + 2(v_2)$

$5 = 5v_1 + 2v_2$ — 10

NEL $v_1 - v_2 = -e(u_1 - u_2)$

$= -\frac{3}{4}(3+5)$

$= -6$ — 10

$v_1 = -1$ m/s and $v_2 = 5$ m/s — 10

(ii) KE before collision $= \frac{1}{2}(5)(3)^2 + \frac{1}{2}(2)(-5)^2$

$= 47.5$ — 5

KE after collision $= \frac{1}{2}(5)(-1)^2 + \frac{1}{2}(2)(5)^2$

$= 27.5$ — 5

KE lost $= 47.5 - 27.5$

$= 20$ J — 5

(iii) Impulse $= |(2)(-5) - (2)(5)|$

$= 20$ Ns — 5

50

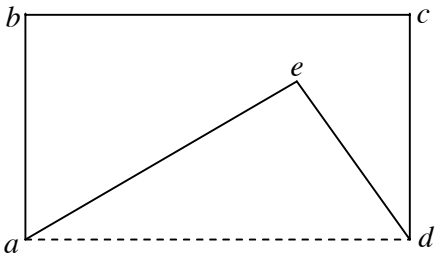
6. **(a)** Particles of weight 4 N, 5 N, 3 N and 2 N are placed at the points (11, 5), (p, q), (-4, 1) and (7, p), respectively.

The co-ordinates of the centre of gravity of the system are (4, q).

Find **(i)** the value of p

(ii) the value of q.

(b) A rectangular lamina with vertices a, b, c and d has the triangular portion with vertices a, d and e removed.



The co-ordinates of the points are $a(0, 0)$, $b(0, 8)$, $c(12, 8)$, $d(12, 0)$ and $e(9, 6)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.

(a)
$$4 = \frac{4(11)+5(p)+3(-4)+2(7)}{14} \qquad 10$$
$$p = 2 \qquad 5$$
$$q = \frac{4(5)+5(q)+3(1)+2(2)}{14} \qquad 10$$
$$q = 3 \qquad 5$$

(b)

	area :	c.g.	
adcb	$(12)(8) = 96$	$(6, 4)$	5
aed	$\frac{1}{2}(12)(6) = 36$	$(7, 2)$	5
lamina	$= 60$	(x, y)	

$$(60)(x) = 96(6) - 36(7) \qquad 5$$
$$x = 5.4$$
$$(60)(y) = 96(4) - 36(2) \qquad 5$$
$$y = 5.2$$

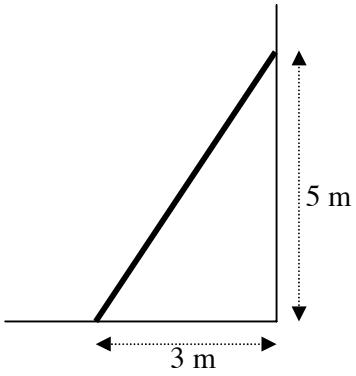
co - ords of c.g. $(5.4, 5.2)$ 50

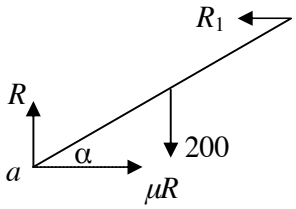
7. **(a)** A uniform ladder, of weight 200 N, rests on rough horizontal ground and leans against a smooth vertical wall.

The foot of the ladder is 3 m from the wall and the top of the ladder is 5 m above the ground.

The ladder is in equilibrium and is on the point of slipping.

Find the coefficient of friction between the ladder and the ground.





horiz $\mu R = R_1$

vert $R = 200$

Take moments about a :

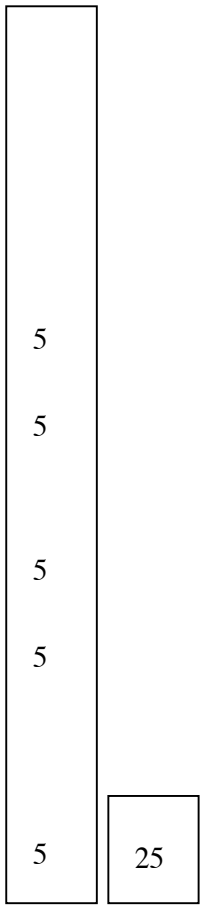
$$R_1(5) = 200(1.5)$$

$$R_1 = 60$$

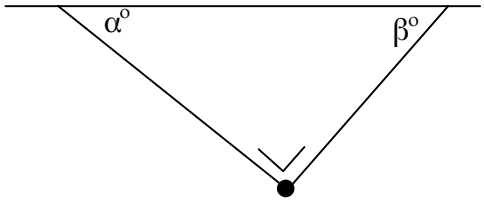
$$\mu R = R_1$$

$$\mu(200) = 60$$

$$\Rightarrow \mu = \frac{3}{10}$$



7 **(b)** Two light inextensible strings are tied to a particle weighing 50 N.



The other ends of the strings are tied to two points on a horizontal ceiling.

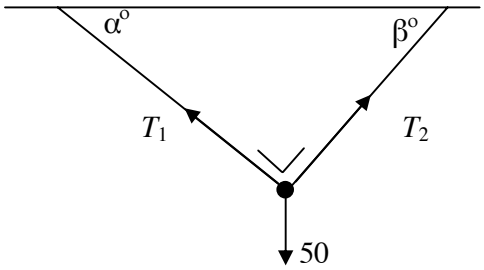
The strings make angles α° and β° with the ceiling, as shown in the diagram.
$\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

(i) Show on a diagram the forces acting on the particle.

(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.

(iii) Solve these equations to find the tension in each of the strings.

(i)



5

(ii) horiz $T_1\cos\alpha = T_2\cos\beta$ 5

vert $T_1\sin\alpha + T_2\sin\beta = 50$ 5

(iii) $T_1\cos\alpha = T_2\cos\beta$

$T_1\left(\frac{3}{5}\right) = T_2\left(\frac{4}{5}\right)$

$T_1 = \left(\frac{4}{3}\right)T_2$ 5

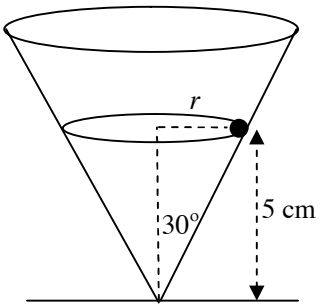
$T_1\sin\alpha + T_2\sin\beta = 50$

$\left(\frac{4T_2}{3}\right)\left(\frac{4}{5}\right) + T_2\left(\frac{3}{5}\right) = 50$

$T_2 = 30$ N and $T_1 = 40$ N 5 25

8. **(a)** A particle describes a horizontal circle of radius 0.5 m with u niform angular velocity ω radians per second.
Its acceleration is 8 m/s^2.
Find **(i)** the value of ω
(ii) the time taken to complete one revolution.

(b) A right circular hollow cone is fixed to a horizontal surface.
Its semi-vertical angle is 30° and its axis is vertical.



A smooth particle of mass 2 kg describes a horizontal circle of radius r cm on the smooth inside surface of the cone.

The plane of the circular motion is 5 cm above the horizontal surface.
(i) Find the value of r in surd form.
(ii) Show on a diagram all the forces acting on the particle.
(iii) Find the reaction force between the particle and the surface of the cone.
(iv) Calculate the angular velocity of the particle.

(a)

(i) $r\omega^2 = \text{acceleration}$

$0.5\omega^2 = 8$

$\Rightarrow \quad \omega = 4 \text{ rad/s}$ — 10

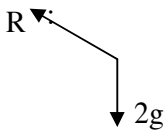
(ii) $\text{Period} = \dfrac{2\pi}{\omega}$

$= \dfrac{\pi}{2}$ s. — 10

(b)

(*i*) $\tan 30 = \dfrac{r}{5} \Rightarrow r = \dfrac{5}{\sqrt{3}}$ cm — 5

(*ii*)



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(*iii*) $R\sin 30 = 2g$

$R\left(\dfrac{1}{2}\right) = 20 \Rightarrow R = 40$ N — 10

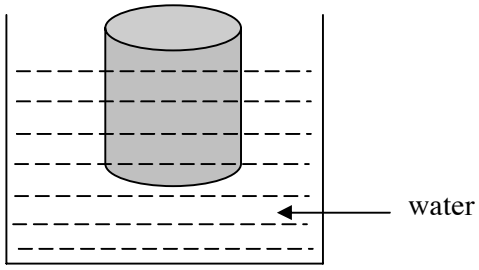
(*iv*) $R\cos 30 = mr\omega^2$

$(40)\left(\dfrac{\sqrt{3}}{2}\right) = 2\left(\dfrac{5}{100\sqrt{3}}\right)\omega^2$

$\Rightarrow \omega = \sqrt{600}$ rad/s — 10

50

9. **(a)** A right circular solid cylinder floats at rest in water with its axis vertical.

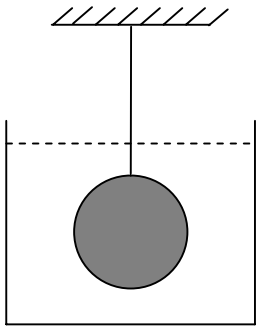


The radius of the cylinder is 6 cm and its height is 20 cm.

75% of the cylinder lies below the surface of the water.

Find the weight of the cylinder.

(b) A solid sphere of radius 7 cm and relative density 3 is completely immersed in a liquid of relative density 0.8.



The sphere is held at rest by a light inelastic vertical string which is tied to a fixed support.

Find the tension in the string.

[Density of water = 1000 kg/m^3].

(a)

$W = B$ 5

W = weight of water displaced

$= \rho V g$

$= 1000\left\{\frac{3}{4}\times\pi\times(0.06)^2\times 0.2\right\}(10)$ 5,5

$\Rightarrow W = 16.96$ N 5

(b)

$B + T = W$ 5

$\frac{W(0.8)}{3} + T = W$ 5

$T = \frac{11}{15}W$ 5

$T = \frac{11}{15}\{3000\}\left\{\frac{4}{3}\times\pi\times(0.07)^3\right\}g$ 5,5

$T = 31.62$ N 5 50

Coimisiún na Scrúduithe Stáit

Marcanna Breise as ucht freagairt trí Gha eilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

Tábla 300 @ 5%

Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 300 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 225 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bonais
226	11
227 - 233	10
234 - 240	9
241 - 246	8
247 - 253	7
254 - 260	6

Bunmharc	Marc Bonais
261 - 266	5
267 - 273	4
274 - 280	3
281 - 286	2
287 - 293	1
294 - 300	0